

# Nonlinear Control of New Single-Phase to Three-Phase Hybrid UPS System

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**Abstract**— This work deals with the problems of uninterruptible power supplies (UPS) based on New Single-Phase to Three-Phase Hybrid UPS System with reduced number of switches built in two stages: an input half-bridge rectifier and an output inverter. The two blocks are joined by a continuous intermediate bus. The objective of control is threefold: i) correcting the power factor "PFC", ii) regulating the DC bus voltage, iii) generation of a symmetrical three-phase system at the output even if the load is unsymmetrical. The synthesis of the controllers has been reached by the technical nonlinear backstepping control. A detailed stability analysis system was performed. The performances of regulators have been validated by numerical simulation in MATLAB / SIMULINK.

**Keywords**—UPS, Backstepping, Nonlinear Control, PFC, Stability.

## I. INTRODUCTION

Uninterruptible power supply (UPS) systems provide uninterrupted, reliable, and high-quality power for vital loads. They, in fact, protect sensitive loads against power outages as well as overvoltage and undervoltage conditions. UPS systems also suppress line transients and harmonic disturbances. Applications of UPS systems include medical facilities, life support systems, data storage and computer systems, emergency equipment, telecommunications and industrial processing.

The advances in power electronics during the past two decades have resulted in a great variety of new topologies and control strategies of UPS systems [1-5]. The issue of reducing the cost of converters has recently been attracting the attention of researchers [6-7].

In this paper, we develop non-linear control for New Single-Phase to Three-Phase Hybrid UPS System with reduced number of switches (Figure 1), which is based on half-bridge converter topology. Its consists of an input inductor L and three IGBT-diode PWM control regulators to ensure the regulation of the component DC voltage output of rectifier, The power factor correction and generating a

symmetrical three-phase system. To achieve these objectives three control loops are used. The first inner loop is designed so that the input current is sinusoidal and in phase with the supply voltage, a unity power factor is guaranteed. The second inner loop is designed such that the inverter generating a symmetrical three-phase system whose the reference signals for the two inverter legs are phase shifted  $120^\circ$  from each other and have the same amplitude and frequency of the signal  $v_a(t)$ . The purpose of the outer loop is adjusted  $\beta$  in such way that the DC component of the converter output voltage coincide with the desired changes in spite of the load setpoint. The control law is synthesized using nonlinear control applying backstepping technique. We must ensure at all times that the overall stability of the closed-loop system is achieved [8-10].

This paper is organized as follows: Section 2 is devoted to the description of New Single-Phase to Three-Phase Hybrid UPS System with reduced number of switches and its model, the synthesis of controllers is developed in section 3 and discussed in section 4. The closed loop performances are illustrated by simulation in Section 5. The conclusion ends the paper.

## II. DESCRIPTION AND MODELING OF SYSTEM

The proposed new UPS system, shown in Figure 1, the input switch is on and the power is passed directly from the AC line to load in phase A. At the same time, the first IGBT leg with switches S1 and S2 works as a half-bridge rectifier and supplies the DC-link bus with power. The other two IGBT legs work in inverter mode the same way as the back-end inverter and feed the loads connected in phases B and C.

The three legs of the system are controlled by three PWM generators generating three binary control signals:  $\mu$  for controlling the pair of switches  $(S_1, S_2)$ ,  $\mu_1$  for the pair  $(S_3, S_4)$  and  $\mu_2$  for the pair  $(S_5, S_6)$ . These are defined as follows:

$$\mu = \begin{cases} 1 & \text{if } S_1 \text{ is ON and } S_2 \text{ is OFF} \\ -1 & \text{if } S_2 \text{ is ON and } S_1 \text{ is OFF} \end{cases}$$

$$\mu_1 = \begin{cases} 1 & \text{if } S_3 \text{ is ON and } S_4 \text{ is OFF} \\ -1 & \text{if } S_4 \text{ is ON and } S_3 \text{ is OFF} \end{cases}$$

$$\mu_2 = \begin{cases} 1 & \text{if } S_5 \text{ is ON and } S_6 \text{ is OFF} \\ -1 & \text{if } S_6 \text{ is ON and } S_5 \text{ is OFF} \end{cases}$$

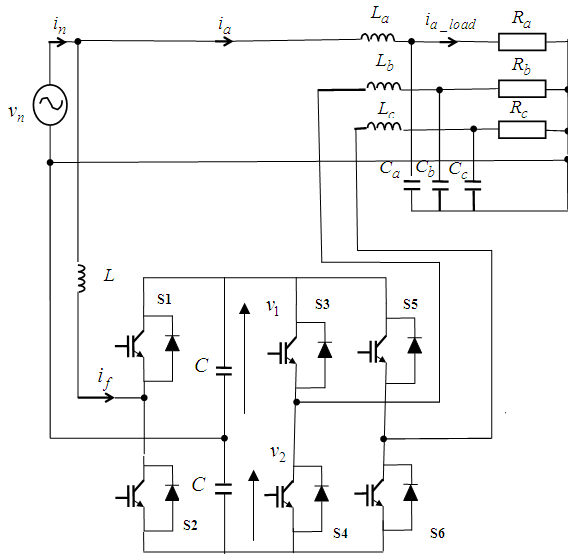


Figure 1. Single-Phase to Three-Phase Hybrid UPS System.

Switched model of the New Single-Phase to Three-Phase Hybrid given by Figure 1, can be obtained using simply the standard Kirchhoff's laws. So doing, one has:

$$L \frac{di_f}{dt} = v_n - (1 + \mu) \frac{v_1}{2} + (1 - \mu) \frac{v_2}{2} \quad (1a)$$

$$C \frac{dv_1}{dt} = (1 + \mu) \frac{i_f}{2} - (1 + \mu_1) \frac{i_b}{2} - (1 + \mu_2) \frac{i_c}{2} \quad (1b)$$

$$C \frac{dv_2}{dt} = -(1 - \mu) \frac{i_f}{2} + (1 - \mu_1) \frac{i_b}{2} + (1 - \mu_2) \frac{i_c}{2} \quad (1c)$$

$$L_a \frac{di_a}{dt} = v_n - v_a \quad (1d)$$

$$L_b \frac{di_b}{dt} = -v_b + (1 + \mu_1) \frac{v_1}{2} - (1 - \mu_1) \frac{v_2}{2} \quad (1e)$$

$$L_c \frac{di_c}{dt} = -v_c + (1 + \mu_2) \frac{v_1}{2} - (1 - \mu_2) \frac{v_2}{2} \quad (1f)$$

$$C_a \frac{dv_a}{dt} = i_a - \frac{1}{R_a} v_a \quad (1g)$$

$$C_b \frac{dv_b}{dt} = i_b - \frac{1}{R_b} v_b \quad (1h)$$

$$C_c \frac{dv_c}{dt} = i_c - \frac{1}{R_c} v_c \quad (1i)$$

The model (1) is useful for building up an accurate simulator of the Single-Phase to Three-Phase Hybrid. However, it cannot be based upon in the control design as it involves binary control inputs, namely  $\mu$ ,  $\mu_1$  and  $\mu_2$ . This kind of difficulty is generally coped with by resorting to average models. Signal averaging is performed over cutting intervals (e.g. Abouloifa et al., 2003). The obtained average model is the following:

$$L \dot{x}_1 = v_n - \frac{1}{2} x_8 u - \frac{1}{2} x_9 \quad (2a)$$

$$C_b \dot{x}_2 = x_3 - \frac{1}{R_b} x_2 \quad (2b)$$

$$L_b \dot{x}_3 = -x_2 + \frac{1}{2} x_8 u_1 + \frac{1}{2} x_9 \quad (2c)$$

$$C_c \dot{x}_4 = x_5 - \frac{1}{R_c} x_4 \quad (2d)$$

$$L_c \dot{x}_5 = -x_4 + \frac{1}{2} x_8 u_2 + \frac{1}{2} x_9 \quad (2e)$$

$$C_a \dot{x}_6 = x_7 - \frac{1}{R_a} x_6 \quad (2f)$$

$$L_a \dot{x}_7 = -x_6 + v_n \quad (2g)$$

$$C \dot{x}_8 = u x_1 - u_1 x_3 - u_2 x_5 \quad (2h)$$

$$C \dot{x}_9 = x_1 - x_3 - x_5 \quad (2i)$$

where the state variables  $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, u, u_1$  and  $u_2$  denote respectively the average values, over a cutting period, of variables  $i_f, v_b, i_b, v_c, i_c, v_a, i_a, v_1 + v_2, v_1 - v_2, \mu, \mu_1$  and  $\mu_2$ .

### III. CONTROLLER DESIGN

#### A. Control Objectives

Control objectives considered for this class of Single-Phase to Three-Phase Hybrid are three:

- The half-bridge rectifier must operate with a power factor close to unity. This is achieved by ensuring that, in equilibrium regime, the current drawn by the converter should be, in average, equal the reference signal defined by  $x_1^* = \beta v_n(t) - x_7$ .
- The DC component of the voltage  $v_1 + v_2$  must be stabilized to a desired reference voltage namely  $x_8^*$ .
- The three-phase DC/AC inverter must generate a symmetrical three-phase system whose the reference signals for the two inverter legs are phase shifted  $120^\circ$  from each other and have the same amplitude and frequency of the signal  $x_6$ .

The proposed control system has the structure shown in Fig. 2. Both controllers 1 and 2 will be synthesized using the backstepping approach and the third will be done by a version of proportional-integral corrector.

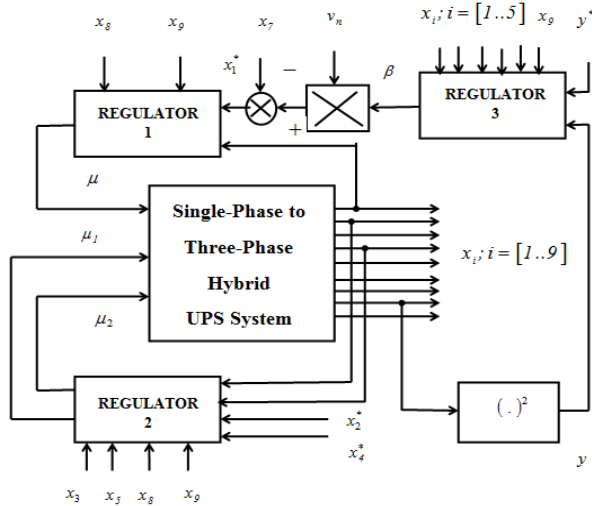


Figure 2. Block diagram of regulators

### B. Current inner loop design (regulator 1)

The control objective is to force the current  $x_1$  to follow the desired reference signal  $x_1^* = \beta v_n(t) - x_7$ , which  $\beta$  is a positive real signal to be defined later. To achieve this, we propose the control backstepping approach.

we introduce the following tracking error on the current  $x_1$ :

$$e_1 = L(x_1 - x_1^*) \quad (3)$$

where  $x_1^*$  denotes the reference signal.

Using equation (2a), time derivative of the equation (3) relates the following error dynamics:

$$\dot{e}_1 = v_n - \frac{1}{2}x_8u - \frac{1}{2}x_9 - L\frac{dx_1^*}{dt} \quad (4)$$

The actual control variable, namely  $u$ , has emerged for the first time in equation (4). An appropriate control law for generating  $u$  must now be determined so that the  $e_1$ -system is made globally asymptotically stable. To this end, consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2}e_1^2 \quad (5)$$

Using (4) and (5), the time-derivative of  $V_1$  can be written as:

$$\dot{V}_1 = e_1 \left( v_n - \frac{1}{2}x_8u - \frac{1}{2}x_9 - L\beta\dot{v}_n + L\dot{x}_7 \right) \quad (6)$$

This shows that, for the  $e_1$ -system to be globally asymptotically stable, it is sufficient to choose the control  $u$  so that  $\dot{V}_1 = -d_1e_1^2$  which, due to (6), amounts to ensuring that:

$$\dot{e}_1 = -d_1e_1 \quad (7)$$

where  $d_1$  is a positive constant synthesis.

Comparing (7) and (4) yields the following backstepping control law:

$$u = \frac{2}{x_8} \left( d_1e_1 + \left( 1 + \frac{L}{L_a} \right) v_n - \frac{L}{L_a}x_6 - \frac{1}{2}x_9 - L\beta\dot{v}_n \right) \quad (8)$$

**Proposition 1.** Consider the Single-Phase to Three-Phase Hybrid given by Fig 1, which is described by the average model (2a). If the first derivative of  $\beta$  is available, then the control law (8) guarantees asymptotic stability of the error signal  $e_1$ .

### C. Three-phase system inner loop design (regulator 2)

The controller must force the three-phase system of voltage of three-phase DC/AC inverter to follow the reference signals  $x_2^* = x_6 \left( t - \frac{T}{3} \right)$  and  $x_4^* = x_6 \left( t - \frac{2T}{3} \right)$ .

The study will be developed for any non-symmetrical resistive load connected to the output of the converter, Regulator synthesis is terminated in two steps since the

relative degree of the two subsystems described respectively by the equations [(2b), (2c)] and [(2d), (2e)] is equal to two.

*Step 1: Stabilization of the subsystem  $(e_2, e_4)$*

Let us introduce the error vector of the two voltages  $x_2$  and  $x_4$  is:

$$Z_1 = \begin{pmatrix} e_2 \\ e_4 \end{pmatrix} = \begin{pmatrix} C_b(x_2 - x_2^*) \\ C_c(x_4 - x_4^*) \end{pmatrix} \quad (9)$$

Using (2b) and (2d) time-derivation of (9) yields the following error dynamics:

$$\dot{Z}_1 = \begin{pmatrix} \dot{e}_2 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} x_3 - \frac{x_2}{R_b} - C_b \dot{x}_2^* \\ x_5 - \frac{x_4}{R_c} - C_c \dot{x}_4^* \end{pmatrix} \quad (10)$$

In (10),  $x_3$  and  $x_5$  stands for the (virtual) control variables. Then,  $e_2$  and  $e_4$  can be regulated to zero respectively if  $x_3 = x_3^*$  and  $x_5 = x_5^*$  where  $X^*$ , called stabilizing function vector, is defined by:

$$X^* = \begin{pmatrix} x_3^* \\ x_5^* \end{pmatrix} = \begin{pmatrix} -d_2 e_2 + \frac{x_2}{R_b} + C_b \dot{x}_2^* \\ -d_4 e_4 + \frac{x_4}{R_c} + C_c \dot{x}_4^* \end{pmatrix} \quad (11)$$

where  $(d_2, d_4)$  are design parameters.

If  $x_3$  and  $x_5$  were actually the controls in (10), imply that  $\dot{Z}_1 = \begin{pmatrix} \dot{e}_2 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -d_2 e_2 \\ -d_4 e_4 \end{pmatrix}$ : which clearly establishes asymptotic stability of (10) with respect to the Lyapunov function:

$$W_1 = 0.5e_2^2 + 0.5e_4^2 \quad (12)$$

Then, time-derivation of  $W_1$  would be:

$$\dot{W}_1 = -d_2 e_2^2 - d_4 e_4^2 < 0 \quad (13)$$

As  $x_3$  and  $x_5$  are not the actual controls in (10), one cannot let  $x_3 = x_3^*$  and  $x_5 = x_5^*$ . However, we retain the

expression of the stabilizing function vector  $X^*$  and introduce a new error vector, between the virtual controls and its desired.

$$Z_2 = \begin{pmatrix} e_3 \\ e_5 \end{pmatrix} = \begin{pmatrix} x_3 - x_3^* \\ x_5 - x_5^* \end{pmatrix} \quad (14)$$

Then, (10) becomes, using (11) and (14)

$$\dot{Z}_1 = \begin{pmatrix} \dot{e}_2 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -d_2 e_2 + e_3 \\ -d_4 e_4 + e_5 \end{pmatrix} \quad (15)$$

Also, the derivative of Lyapunov function (13) becomes:

$$\dot{W}_1 = -d_2 e_2^2 + e_2 e_3 - d_4 e_4^2 + e_4 e_5 \quad (16)$$

This completes the first step.

*Step 2: Stabilization subsystem  $(e_2, e_3, e_4, e_5)$*

Time-derivation of  $Z_2$  gives, using (2c), (2e) and (14)

$$\dot{Z}_2 = \begin{pmatrix} \dot{e}_3 \\ \dot{e}_5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{L_b} x_2 + \frac{1}{2L_b} x_8 u_1 + \frac{1}{2L_b} x_9 - \dot{x}_3^* \\ -\frac{1}{L_c} x_4 + \frac{1}{2L_c} x_8 u_2 + \frac{1}{2L_c} x_9 - \dot{x}_5^* \end{pmatrix} \quad (17)$$

The actual control variables, namely  $u_1$  and  $u_2$ , appears for the first time in (17). An appropriate control laws for generating  $u_1$  and  $u_2$  has now to be found for the system (10) and (17) whose state vector is  $(e_2, e_3, e_4, e_5)$ . Let us consider the Lyapunov function  $W_2$ .

$$W_2 = W_1 + 0.5e_3^2 + 0.5e_5^2 \quad (18)$$

Using (16), the time-derivative of  $W_2$  can be rewritten as:

$$\dot{W}_2 = -d_2 e_2^2 + e_3(e_2 + \dot{e}_3) - d_4 e_4^2 + e_5(e_4 + \dot{e}_5) \quad (19)$$

This shows that, for the  $(e_2, e_3, e_4, e_5)$ -system to be globally asymptotically stable, it is sufficient to choose the control  $u_1$  and  $u_2$  so that  $\dot{W}_2 = -d_2 e_2^2 - d_3 e_3^2 - d_4 e_4^2 - d_5 e_5^2$  which, due to (19), amounts to ensuring that:

$$\dot{Z}_2 = \begin{pmatrix} \dot{e}_3 \\ \dot{e}_5 \end{pmatrix} = \begin{pmatrix} -e_2 - d_3 e_3 \\ -e_4 - d_5 e_5 \end{pmatrix} \quad (20)$$

Comparing (20) and (17) yields the following backstepping control laws  $u_1$  and  $u_2$ :

$$u_1 = \frac{2}{x_8} \left( x_2 - \frac{1}{2} x_9 + L_b \dot{x}_3^* - L_b e_2 - L_b d_3 e_3 \right) \quad (21)$$

$$u_2 = \frac{2}{x_8} \left( x_4 - \frac{1}{2} x_9 + L_c \dot{x}_5^* - L_c e_4 - L_c d_5 e_5 \right) \quad (22)$$

The results thus established are summarized in the following proposition.

**Proposition 2.** Consider the control system, next called inner closed-loop, consisting of the two subsystems (2b)-(2c), (2d)-(2e) and the control laws (21), (22). One has the following.

The inner closed-loop system undergoes the following equation in the  $(e_2, e_3, e_4, e_5)$ -coordinates

$$\begin{pmatrix} \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \end{pmatrix} = \begin{pmatrix} -d_2 & 1 & 0 & 0 \\ -1 & -d_3 & 0 & 0 \\ 0 & 0 & -d_4 & 1 \\ 0 & 0 & -1 & -d_5 \end{pmatrix} \begin{pmatrix} e_2 \\ e_3 \\ e_4 \\ e_5 \end{pmatrix} \quad (23)$$

Furthermore, (23) is globally asymptotically stable with respect to the Lyapunov function  $W_2 = W_1 + 0.5e_3^2 + 0.5e_5^2$  because  $\dot{W}_2 = -d_2 e_2^2 - d_3 e_3^2 - d_4 e_4^2 - d_5 e_5^2$  is negative definite. As (23) is linear, then the error vector converges exponentially fast to zero, whatever the initial conditions. It follows in particular that the (average) three-phase system of voltage tends asymptotically (and exponentially fast) to its reference signals  $x_2^* = x_6 \left( t - \frac{T}{3} \right)$  and  $x_4^* = x_6 \left( t - \frac{2T}{3} \right)$ .

#### D. DC Bus voltage outer loop design (regulator 3)

- Relation between  $\beta$  and  $x_8$

The aim of the outer loop is to generate a tuning law for the signal  $\beta$  so that the output voltage  $x_8$  is steered to a given reference value  $x_8^*$ .

**Hypothesis:** The inner loops of the input current (regulator 1) and the Three-phase system (regulator 2) are

assumed to have fast dynamics relative the outer loop of DC voltage (regulator 3).

The first step in designing such a loop is to establish a relationship between  $\beta$  (the control signal) and the squared output voltage  $y = x_8^2$ . This is the subject of the following proposition.

**Proposition 3.** Consider the Single-Phase to Three-Phase Hybrid described by (2a)-(2i) augmented with the inner control laws defined by (8), (21) and (22). Under the same assumptions as in proposition 1 and proposition 2, one has the following:

The squared-voltage  $y = x_8^2$  varies, in response to the tuning ratio  $\beta$ , according to the following first-order time-varying linear equation:

$$\dot{y} = f(\beta) + p(t) + q(x_i) \quad (24)$$

with

$$f(\beta) = k_0 \beta \quad ; \quad p(t) = -k_0 \beta \cos(2\omega t) - k_1 \beta^2 \sin(2\omega t)$$

$$q(x_i) = \frac{4}{C} \left[ L \beta v_{ir} v_i - \left( 1 + \frac{L}{L_a} \right) v_{ir} v_i - \frac{L}{L_a} x_6^* x_1 - \frac{1}{2} x_6^* x_2^* - \frac{1}{2} x_6^* x_3^* - L_b x_3^* x_5^* - L_b x_5^* x_3^* - \frac{1}{2} x_6^* x_4^* - L_c x_5^* x_5^* \right]$$

where

$$k_0 = \frac{2E^2}{C} \left( 1 + \frac{L}{L_a} \right) \quad ; \quad k_1 = \frac{2E^2}{C} L \omega$$

**Proof:** Based on this hypothesis, the control laws (8), (21) and (22) are reduced to:

$$u = \frac{2}{x_8} \left[ \left( 1 + \frac{L}{L_a} \right) v_n - \frac{L}{L_a} x_6 - \frac{1}{2} x_9 - L \beta \dot{v}_n \right]$$

$$\text{with} \quad v_n = E \sin(\omega t)$$

$$u_1 = \frac{2}{x_8} \left( x_2 - \frac{1}{2} x_9 + L_b \dot{x}_3^* \right) \quad \text{and} \quad u_2 = \frac{2}{x_8} \left( x_4 - \frac{1}{2} x_9 + L_c \dot{x}_5^* \right)$$

Referring equations  $u$ ,  $u_1$  and  $u_2$  in equation (2h), we obtain:

$$C\dot{x}_8 = \frac{2}{x_8} \left[ \left( 1 + \frac{L}{L_u} \right) v_n - \frac{L}{L_u} x_6 - \frac{1}{2} x_9 - L\beta v_n \right] x_7 - \frac{2}{x_8} \left[ x_2 - \frac{1}{2} x_9 + L_b x_3^* \right] x_3^* - \frac{2}{x_8} \left[ x_4 - \frac{1}{2} x_9 + L_c x_5^* \right] x_5^* \quad \begin{cases} \dot{e}_6 = -d_6 e_6 - d_7 e_7 - p(t) \\ \dot{e}_7 = e_6 \end{cases} \quad (26)$$

with  $x_7^* = \beta v_n(t) - x_7$

Implies that:

$$C\dot{x}_8 x_8 = 2 \left[ \left( 1 + \frac{L}{L_u} \right) v_n x_7^* - L\beta v_n x_7^* \right] - 2 \left[ \frac{L}{L_u} x_6 x_7^* - \frac{1}{2} x_9 x_7^* + x_2 x_3^* - \frac{1}{2} x_9 x_3^* \right] - 2 \left[ L_b x_3^* x_3^* + x_4 x_5^* - \frac{1}{2} x_9 x_5^* + L_c x_5^* x_5^* \right]$$

If is lying  $y = x_8^2$ , its derivative is:

$$\dot{y} = \frac{4E^2}{C} \left[ \left( 1 + \frac{L}{L_u} \right) \beta \sin^2(\omega t) - \frac{1}{2} L\alpha \beta^2 \sin(2\omega t) \right] - \frac{4}{C} \left[ L\beta v_n x_7 - \left( 1 + \frac{L}{L_u} \right) v_n x_7 - \frac{L}{L_u} x_6 x_7^* \right] - \frac{4}{C} \left[ \frac{1}{2} x_9 x_3^* + x_2 x_3^* - \frac{1}{2} x_9 x_3^* + L_b x_3^* x_3^* + x_4 x_5^* - \frac{1}{2} x_9 x_5^* + L_c x_5^* x_5^* \right]$$

Finally, we obtain the equation (24) and complete the proof of Proposition 3

- Squared output voltage control

The signal  $f(\beta)$  is considered as a (virtual) control input of the system. The term  $p(t)$  is treated as a perturbation. The current problem is to design a suitable control law so that the square of the voltage  $y = x_8^2$  follows a reference signal given  $y^* = (x_8^*)^2$ . As the term disruptive  $p(t)$  in (25), is periodic with zero mean and a control law PI with compensation for non-linearity  $q(x_i)$ , should suffice:

$$f(\beta) = d_6 e_6 + d_7 e_7 - q(x_i) + y^* \quad (25)$$

with  $e_6 = y^* - y$  ;  $e_7 = \int_0^t e_6 d\tau$

Substituting equation (25) in (24), we obtain the Equations errors  $e_6$  and  $e_7$ .

At this point, the regulator parameters  $(d_6, d_7)$  are any positives real constants. The following analysis will show clearly how they should be chosen so that the control objectives are achieved.

The actual control signal  $\beta$  can be easily obtained from (25) using the fact that  $f^{-1}(\cdot)$  exists.

#### IV. ANALYSIS OF CLOSED LOOP STABILITY

By combining (7), (15), (20) and (26), we obtain the following equation describing the evolution of the state vector, denoted  $E_r = (e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6 \ e_7)^T$ :

$$\dot{E}_r = AE_r + P \quad (27)$$

where

$$A = \begin{pmatrix} -d_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -d_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -d_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_4 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -d_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -d_6 & -d_7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$P = (0 \ 0 \ 0 \ 0 \ 0 \ -p(t) \ 0)^T$$

The stability of the above system will now be analyzed using the averaging theory. Now introduce the time-scale change  $\tau = \omega t$ . It is readily seen from (27) that  $Z(\tau) \equiv E_r(\tau / \omega)$  undergoes the differential equation:

$$\frac{dZ(\tau)}{d\tau} = \varepsilon AZ(\tau) + \varepsilon P_\tau(\tau) \quad (28)$$

where

$$P_\tau(\tau) = (0 \ 0 \ 0 \ 0 \ 0 \ -p(\tau/\omega) \ 0)^T$$

Now, let us introduce the average functions:

$$\bar{Z} = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\pi} \int_0^{2\pi} Z(\tau) d\tau$$

$$\bar{P}_\tau = \lim_{\varepsilon \rightarrow 0} \frac{1}{2\pi} \int_0^{2\pi} P_\tau(\tau) d\tau$$

It follows from (28) that:

$$\bar{P}_\tau = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \quad (29)$$

In order to get stability results regarding the system of interest (27), it is sufficient (thanks to averaging theory) to analyze the following averaged system:

$$\dot{\bar{Z}} = \varepsilon A \bar{Z} \quad (30)$$

To this end, notice that (30) has a unique equilibrium at:

$$Z^* = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \quad (31)$$

On the other hand, as (30) is linear, the stability properties of its equilibrium are fully determined by the state-matrix  $A$ . More specifically, the equilibrium  $Z^*$  will be globally exponentially stable if the matrix  $A$  is Hurwitz. To this end, we note that the eigen values are zeros the following characteristic polynomial:

$$\det(\lambda I - A) = \lambda^7 + a_6 \lambda^6 + a_5 \lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda^1 + a_0 \quad (32)$$

where

$$a_6 = d_1 + d_2 + d_3 + d_4 + d_5 + d_7$$

$$a_5 = d_1 d_2 + d_4 d_5 + b_4 (d_4 + d_5) + b_5 (1 + d_1 + d_2)$$

$$a_4 = d_4 d_5 b_4 + (d_1 d_2 + b_5 (1 + d_1 + d_2))(d_4 + d_5) + d_3 d_7 + (d_1 + d_2) b_5 + d_1 d_2 (d_3 + d_6) + d_2 + d_3 - d_4 - d_5 - d_6 + d_7$$

$$a_3 = 1 + b_1 + b_3 - d_7 - d_1 d_6 + d_4 d_5 (b_3 - 1) + (d_4 + d_5)(b_2 - d_1 - d_6)$$

$$a_2 = b_0 + b_2 - d_1 d_7 + b_2 d_4 d_5 + (d_4 + d_5)(b_1 - d_1 d_6 - d_7) + (d_1 + d_6)(1 - d_4 d_5)$$

$$a_1 = b_1 (1 + d_4 d_5) + (b_0 - d_1 d_7)(d_4 + d_5) + (d_1 d_6 + d_7)(1 - d_4 d_5)$$

$$a_0 = b_0 (1 + d_4 d_5) + d_1 d_7 (1 - d_4 d_5)$$

with

$$b_5 = d_3 d_6 + d_7; \quad b_4 = d_1 + d_2 + d_3 + d_7;$$

$$b_3 = d_1 d_2 + (1 + d_1 + d_2) b_5$$

$$b_2 = d_3 d_7 + (d_1 + d_2) b_5 + d_1 d_2 (d_3 + d_6)$$

$$b_1 = d_1 d_2 b_5 + (d_1 + d_2) d_3 d_7; \quad b_0 = d_1 d_2 d_3 d_7$$

The application of famous algebraic Routh criterion implies that the system is stable at the condition that:

$$a_0 > 0; \quad a_6 > 0; \quad a_6 a_5 - a_4 > 0$$

$$c_1 = (a_6 a_5 - a_4) a_4 - (a_6 a_3 - a_2) a_6 > 0$$

$$c_2 = c_1 (a_6 a_3 - a_2) - (a_6 a_5 - a_4) ((a_6 a_5 - a_4) a_2 - (a_6 a_1 - a_0) a_6) > 0$$

$$c_3 = c_2 ((a_6 a_5 - a_4) a_2 - (a_6 a_1 - a_0) a_6) - c_1 ((a_6 a_1 - a_0) c_1 - (a_6 a_5 - a_4) a_0) > 0$$

$$c_4 = c_3 ((a_6 a_1 - a_0) c_1 - (a_6 a_5 - a_4) a_0) - c_2 a_0 > 0$$

The equilibrium  $Z^*$  of the linear system (30) is actually globally exponentially stable. Applying e.g. Theorem 4.10 in [11], one concludes that there exists a  $\varepsilon^* > 0$  such that for  $\varepsilon < \varepsilon^*$ , the differential equation (30) has a harmonic solution  $Z = Z(t, \varepsilon)$  that continuously depends on  $\varepsilon$ . Moreover, one has  $\lim_{\varepsilon \rightarrow 0} Z(t, \varepsilon) = Z^*$ . This, together with (31), yields in particular that:

$$\lim_{\varepsilon \rightarrow 0} e_1(t, \varepsilon) = 0, \quad \lim_{\varepsilon \rightarrow 0} e_2(t, \varepsilon) = 0, \quad \lim_{\varepsilon \rightarrow 0} e_3(t, \varepsilon) = 0$$

$$\lim_{\varepsilon \rightarrow 0} e_4(t, \varepsilon) = 0, \quad \lim_{\varepsilon \rightarrow 0} e_5(t, \varepsilon) = 0, \quad \lim_{\varepsilon \rightarrow 0} e_6(t, \varepsilon) = 0$$

$$\lim_{\varepsilon \rightarrow 0} e_7(t, \varepsilon) = 0$$

V. NUMERICAL SIMULATIONS

The performances of proposed controllers were validated by simulation in MATLAB/SIMULINK environment. The parameters of the controlled system are given in the table below:

TABLE I. SYSTEM PARAMETERS CONTROLLED

Parameters	Symbol	Values
Network	$E$	$220\sqrt{2} \text{ V}$
	$f$	$50\text{Hz}$
Rectifier	$L$	$2\text{mH}$
DC Bus	$C$	$10\text{mF}$
DC/AC Converter	$L_a = L_b = L_c$	$10\text{mH}$
	$C_a = C_b = C_c$	$100\mu\text{F}$
Regulator 1	$d_1$	1000
Regulator 2	$d_2 = d_4$	1000
	$d_3 = d_5$	500
Regulator 3	$d_6$	$2 * 10^{-6}$
	$d_7$	$6 * 10^{-5}$

The mains voltage is fixed at its nominal value  $v_n(t) = E \sin(\omega t)$  and the reference DC bus voltage changes from 800V to 1000V. Figures 3 to 7 show the simulation results of uninterrupted power system based on the New Single-Phase to Three-Phase Hybrid with reduced number of switches for a nonsymmetrical resistive load :  $R_a = 10\Omega$ ,  $R_b = 15\Omega$  and  $R_c = 20\Omega$ .

Figures 3 show the control signals  $\mu$ ,  $\mu_1$  and  $\mu_2$  are bounded. In Figure 4, we see that the current  $i_n$  and the input voltage  $v_n$  are sinusoidal and in phase. This shows that the correction of the power factor is well established. Figure 5 and 6 shows the evolution of the output phase voltages  $v_a$ ,  $v_b$  and  $v_c$  as well as the output load currents  $i_{a\_load}$ ,  $i_{b\_load}$  and  $i_{c\_load}$ .

Finally, Figure 7 shows that the DC bus voltage  $x_g$  perfectly follows (in average) its reference. In addition, we note that the ripple voltage oscillates at a frequency  $2\omega$ , but its amplitude is too low compared to the average value of signals.

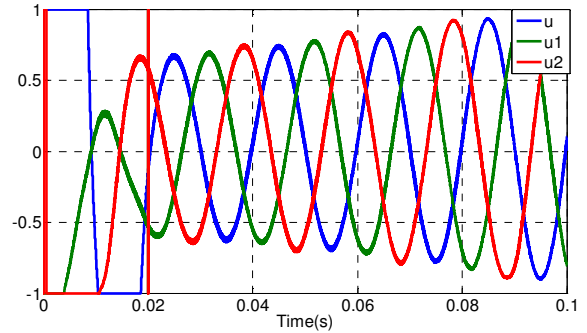


Figure 3. Control signals  $u$ ,  $u_1$  and  $u_2$

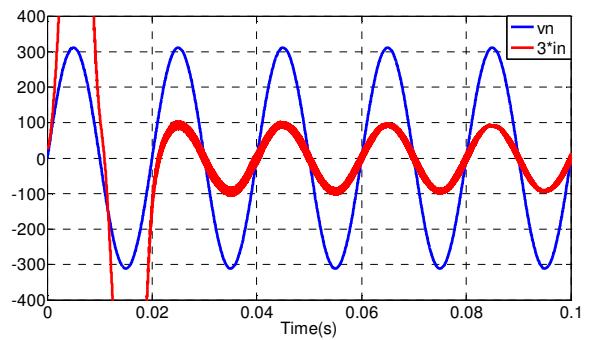


Figure 4. Input voltage  $v_n$  and input current  $3 * i_n$

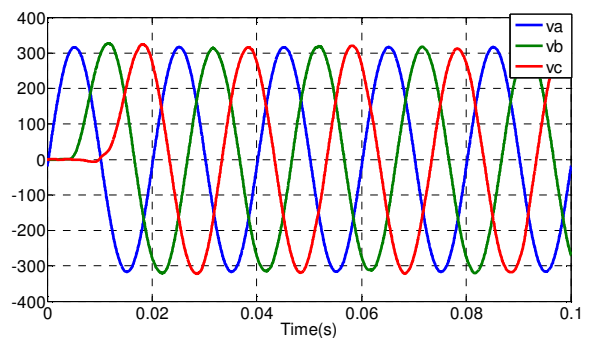
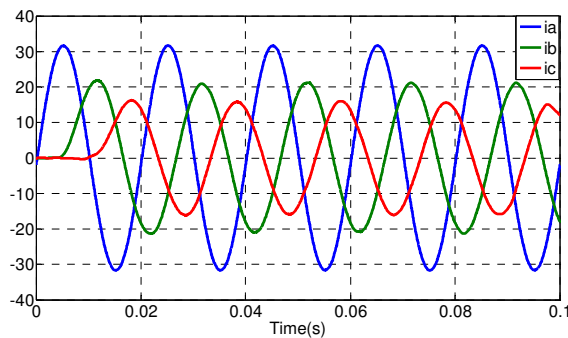
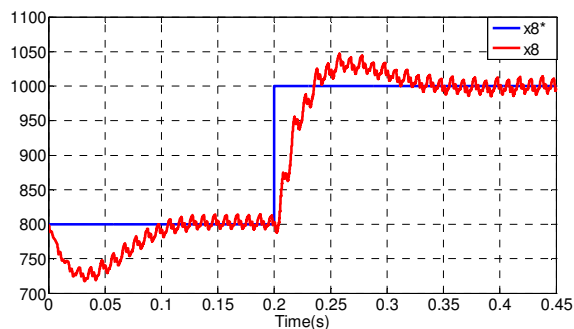


Figure 5. Phase output voltages  $v_a$ ,  $v_b$  and  $v_c$



Figure 6. Load phase current  $i_a$ ,  $i_b$  and  $i_c$ Figure 7. DC bus voltage  $x_g$  and reference  $x_g^*$ 

## VI. CONCLUSION

In this paper, a nonlinear controller is proposed for the New Single-Phase to Three-Phase Hybrid UPS System with reduced number of switches used in power systems without interruption. It has been formally established that the obtained controller meets its objectives such as:

- High-quality sinusoidal output voltages, even with nonsymmetrical loads.
- Unity input power factor PFC feature is enabled.
- Regulation of the DC bus voltage.
- Excellent transient characteristics and stability

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